Source-driven homogenization theory for electro-momentum coupled scatterers

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Willis materials are metamaterials whose subwavelength asymmetry couples the macroscopic pressure-strain and momentum-velocity relations[1]. Recently, the design space of these metamaterials has been expanded to consider additional field coupling such as poroelasticity[2] and piezoelectricity. Of interest in this work is the case of subwavelength asymmetric piezoelectric scatterers in a background material which has been shown to couple the electric field-electric displacement relation to the constitutive equations in the Willis form[3,4]. The existence of this so-called electro-momentum coupling was first predicted using dynamic homogenization of heterogeneous piezoelectric media[1-3] but it can also be understood through a generalized polarizability tensor $\boldsymbol{\alpha}$ that calculates the electro-acoustic field scattered from a single asymmetric piezoelectric inhomogeneity as a function of local fields. We present a multiple-scattering dynamic homogenization method that extends the work of Sieck et al.[5] using the generalized polarizability tensor[6,7]. The model considers the microscale scattered pressure, velocity, and electric fields in a one-dimensional periodic lattice of identical scatterers to find analytical expressions for the effective macroscale fields. The macroscale fields are then used to find the effective electro-momentum coupling constants in terms of the polarizability of the individual scatterers and the concentration of scatterers in a background medium. The resulting expressions for the effective properties obey constraints imposed by reciprocity and passivity and demonstrate an emergent magneto-momentum coupling even in absence of piezomagnetic coupling or electromagnetic bianisotropy at the microscale.

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