Hyperbolic Wave Attractors

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Chaos theory is at the basis of many breakthroughs of the last century, ranging from the understanding of climate change to quantum physics and astronomy. In the context of wave physics, irregularly shaped cavities are known to feature a chaotic response, with rich implications for their dynamics and technological opportunities[1]. At the other extreme, hyperbolic media are characterized by extreme optical anisotropy, supporting highly directional propagation of ray-like waves[2]. Here, we study the intriguing dynamics of oddly-shaped cavities made of hyperbolic media. By leveraging the interplay between the directionality of hyperbolic wave propagation and the broken symmetry of oddly-shaped cavities, we demonstrate the emergence of a broadband, scale-independent and inherently stable wave pattern (Fig. 1a), which we dub a *hyperbolic wave attractor* given its analogy with limit cycles in nonlinear dynamical systems[3], and internal wave attractors occurring in stratified fluids[4].

We demonstrate the unique features of this wave phenomenon using linear elastodynamic waves in a pillared metasurface[5] (Fig. 1b). Notably, we establish a chiral convergence in the attractor formation wherever the source location (Fig. 1c), and the existence of continua of wave patterns with topological features across broad frequency ranges, well-suited to realize a proof-of-concept phonon spectrometer with some robustness to defects. Our findings connect hyperbolic media with attractor physics leveraging tailored broken symmetries at the material and cavity level, with implications for dynamical systems, particle trapping, sensing and topological wave steering, and with potential applications into emerging phononic, photonic and polaritonic platforms.

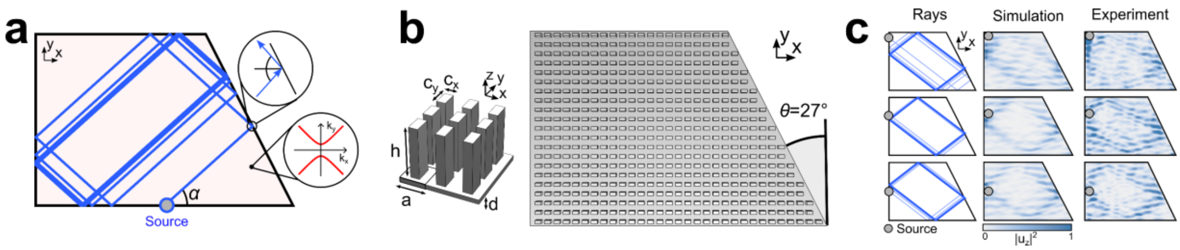

Figure 1. Hyperbolic wave attractor*. *a*, *ray tracing corresponding to the asymptotic behavior of hyperbolic media in a trapezoid cavity, showing a chiral convergence towards a stable geometric pattern: the attractor.* ***b, *sketch of the pillared elastic metasurface.* ***c***, *Same hyperbolic wave attractor pattern obtained with ray tracing, simulation and experiment with different source positions at a fixed frequency.*

References

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