A geometric phase for diffraction of scalar waves

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1. Introduction: geometric phases

In 1984, Berry showed that the phase of waves evolves not just dynamically, but also via adiabatic cyclic transformations of the system [1]. Famously, such an effect in polarization optics was demonstrated by Pancharatnam in 1955 [2], predating Berry's generalized description, now often called the "Pancharatnam-Berry" or "PB" phase. It has had profound impacts on applied science and technologies, especially metasurface optics [3].

Here, we show that another geometric phase predated Berry's description: the detour phase, observed and used in diffractive optics yet conventionally associated with propagation (a dynamic phase). The term was coined by Brown and Lohmann in 1966 [4], and the computer-generated holography community, and more recently the metasurface community, has leveraged it ever since to control the scattering of light. To our knowledge, identification as a geometric phase has eluded the scientific community, likely due to confusion stemming from dispersion.

We demonstrate the geometric nature of the detour phase, establishing its topologically nontrivial nature. We clarify that the detour phase (1) arises from nontransitive transport on a Bloch sphere, associated with Berry curvature; (2) has a geometric component with no dispersion; (3) is a scalar effect (polarization agnostic); (4) can spatially manipulate phase without a geometric discontinuity (impossible with propagation phases); and (5) recontextualizes diffraction observed in many conventional devices. We further argue that the detour phase is *generic* to diffraction and demonstrate its presence in many systems, including aperiodic gratings and nonlocal metasurfaces.

2. Results

Figure 1 directly compares a simple triangular grating (a-d) to a conventional PB phase metasurface (e-h) (as implemented in Ref. [5]). In both systems, the geometries (a,e) are designed to manipulate coupling between two states; leftward and rightward plane waves for the grating, and left-hand and right-hand circular polarization for the metasurface. Linear combinations of these states are placed on a Bloch sphere (b), which is the Poincare sphere in the case of polarization (f). In a suitably defined 2D parameter space defined by two geometrical parameters d_1 and d_2 (c,g), the amplitude and phase of the converted states form a topological feature, i.e., a phase singularity at the origin (d,h). This feature is a hallmark of geometric phases: cyclic evolution of the parameter around the singularity returns to the starting geometry having picked up an integer multiple of 2π.

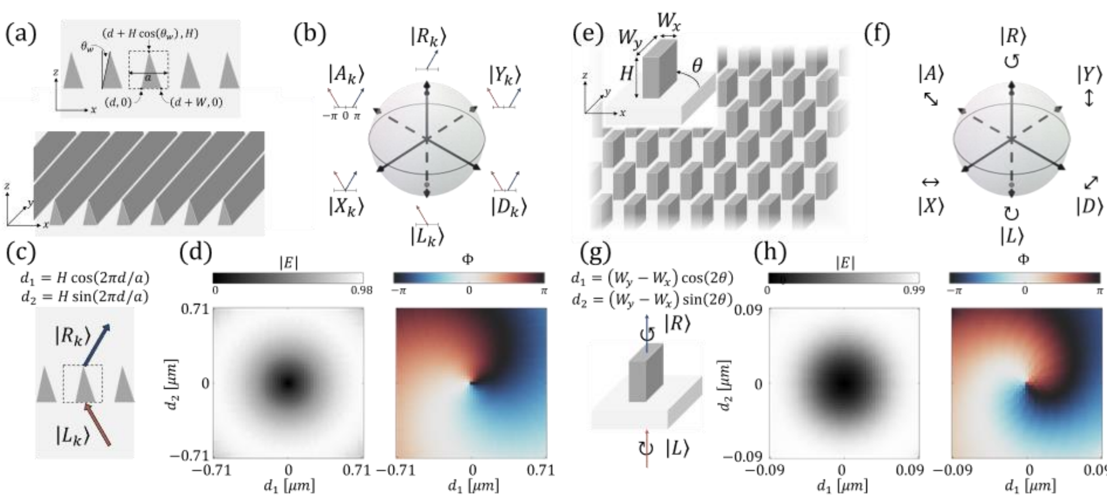

Fig. 1. (a) Geometry for a triangular grating. (b) Bloch sphere for the two states incident at ±30º, shown in (c). (d) Amplitude and phase in the 2D parameter space defined in (c). (e-h) Analogous panels for a PB phase metasurface.

3. References

[1] M.V. Berry, *Proceedings of the Royal Society A.* **392**, 45-57 (1984).
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[4] B.R. Brown and A.W. Lohmann, *Applied Optics* **5**, 967-969 (1966).
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