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Bounds and Limitations on Quasi-Static Passive Cloaking

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Abstract: In this talk, we discuss our recent results [1] on the following challenging question: is it possible to use a passive cloak to make invisible a dielectric inclusion on a finite frequency interval in the quasistatic regime of Maxwell's equations for an observer close to the object? First, we will present our theorem that gives the answer to this question: No one cannot due to the passivity of the cloaking device. Second, we discuss how the passivity assumption allows us to derive sum rules for the Dirichlet-to-Neumann map in the near-field cloaking problem, based on its properties related to two important classes of analytic functions, namely, Herglotz and Stieltjes functions. Third, we present our theorem on bounds that impose fundamental limits on passive cloaking devices over a finite frequency interval. These results extend, to the near-field problem, those of [2] for the far-field problem by combining two techniques for producing bounds: i) variational bounds (using the Dirichlet and Thomson variational principles) from abstract theory of composite framework [3, 4, 5]; ii) the analytic approach to bounds using sum rules for passive systems [2, 6].

References

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