Honeycomb structures perturbed along irrational edges

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In this work, we study wave propagation in two-dimensional continuous honeycomb structures with a line defect. We focus on a Schrödinger operator $H := -\Delta + Q(\boldsymbol{x})$ in $\mathbb{R}^2$ with a perturbed honeycomb potential, where the perturbation consists of a transition, along an edge, between two distinct parity-breaking periodic structures.

For edges with rational slopes, the perturbed medium remains translation-invariant along the edge. In this case, prior work has established the existence of *edge states*, namely time-harmonic solutions of the corresponding wave equation, that propagate along the edge and decay in the transverse direction. For irrational slopes, however, the lack of translation invariance along the edge makes the very definition of edge states nontrivial.

Our approach relies on the observation that the perturbed potential is quasiperiodic along irrational edges, meaning it can be viewed as a slice of a 3D potential with translation-invariance in the direction of the edge. This property allows us to embed our problem in a higher-dimensional framework, defining edge states as restrictions of solutions to an augmented 3D degenerate Schrödinger eigenvalue problem.

Using this framework, we construct a family of approximate solutions to the augmented Schrödinger eigenvalue problem that propagate along the edge and decay in the transverse direction. The asymptotic behavior of these approximate edge states is governed by 1D effective Dirac operators. Unlike for rational edges, the asymptotic analysis leads to infinitely many effective Dirac operators, resulting in infinitely many linearly independent solutions.

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